# ASSESSING THE RESILIENCE OF THE TEXAS POWER GRID NETWORK

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## ABSTRACT

Understanding the structural properties of the power grids under different disruptive event scenarios is the key towards improvement of the security, reliability, and efficiency of modern power systems. In this pilot study, the concepts of topological data analysis, particularly, persistent homology, are used to derive a new metric for resilience of power grid networks. The proposed approach is illustrated in application to a simulated version of the Texas power grid network, under node and edge based attacks for three different weight functions.

***Index Terms*—** Transmission lines, Complex networks, Topological features, Power system resilience.

## 1. INTRODUCTION

Modern power systems have been undergoing a dramatic transformation because of the increasing penetrations of variable renewable energy resources, such as wind power, solar power (including both utility-scale and behind-the-meter solar), and other distributed energy resources [1, 2]. As a result, new cyber-security and resilience vulnerabilities are created that have not been anticipated by the designers of these systems [3]. Hence, while assessment of the modern power system organization becomes increasingly more challenging, there exists an ever growing demand for new accurate and reliable resilience metrics of the energy system under different disruptive event scenarios which could originate from power system component failures, attacks, forecasting errors (e.g., weather or demand forecasts), as well as high impact events such as natural disasters [4, 5, 6, 7, 8].

By further advancing the approach of [9], in this pilot study we investigate the utility of topological data analysis (TDA) and, particularly, persistent homology as new tools for enhancing our understanding of power system resilience under node and edge attacks. We illustrate the proposed geometry-enhanced approach to resilience analysis in application to a simulated version of the Texas power grid network.

## 2. RESILIENCE OF POWER FLOW NETWORKS VIA TOPOLOGICAL DATA ANALYSIS

We begin with a brief overview of relevant concepts.

**Power Grids as Graphs** Define a weighted graph $G = (V, E, \omega)$ as a model for a power grid network, where $V$ is the set of vertices and $E \subset V \times V$ is the set of edges, and $\omega : V \times V \mapsto \mathbb{R}_{\geq 0}$ is the *(edge) weight* such that each edge $e_{uv} \in E$ has a weight $\omega_{uv}$. The edges $e_{uv} \in E$ represents a transmission line between nodes $u$ and $v$, and vertices represent, e.g., power station, substations, transformers. We assume that $G$ is *undirected* i.e., for all $e_{uv} \in E$, $e_{uv} \equiv e_{vu}$. We also assume $e_{uu} \in E$ and $\omega_{uu} = 0$ for all $u \in V$.

**Weighted Power Grid Network and Persistent Homology** Geometry of power grid networks is proven to play an important role behind structure, organization and stability of power systems. One approach to characterize power grid geometry and associated vulnerability properties is via network motifs, i.e., multi-node subgraph patterns [10, 11, 12]. However, motif based analysis of power grid networks is mainly restricted to unweighted graphs and does not allow for straightforward incorporation of important characteristics of power flow networks.

Alternatively, to study the geometry and topology of power flow networks, we can employ the tools of topological data analysis (TDA) [13, 14, 15, 16, 17]. In this study, following [9], we focus on persistent homology and, particularly, Betti numbers [18] as power grid topological summaries due to their flexibility in integration with statistical and machine learning models.

Let $G = (V, E, \omega)$, an (edge)-weighted graph, be a representation of a power grid network. For a certain threshold (or scale) $\nu_j > 0$ and we only keep edges with weights $\omega_{uv} \leq \nu_j$, therefore, we obtain a graph $G_j$ which has the adjacency matrix $A_{uv} = \mathbb{1}_{\omega_{uv} \leq \nu_j}$. We can get a hierarchical nested sequence of graphs $G_1 \subseteq G_2 \subseteq \ldots \subseteq G_n$ for the thresholds

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$\nu_1 < \nu_2 < \ldots < \nu_n$, respectively. This procedure is called *network filtration*.

Next, we identify a simplicial complex with each $G_j, j = 1, \ldots, n$, which is a finite collection of simplices. A $k$-simplex is the convex hull of $k+1$ points. That is, a node is a 0-simplex, an edge is a 1-simplex, a triangle is a 2-simplex, etc. Simplicial complexes allow us to approximate the hidden geometric structure of the power grid in a combinatorial way. Due to computational efficiency [19], the Vietoris-Rips (VR) complex is one of the most popular choices in TDA. For $\nu_j > 0$ the Vietoris-Rips complex is defined as $VR_j = \{\sigma \subset V | \omega_{uv} \leq \nu_j \text{ for all } u, v \in \sigma\}$, i.e., $VR_j$ contains all the $k$-node subsets of $G_j$, $k = 1, \ldots, K$, which are pairwise connected by an edge as simplices of dimension $k-1$.

Now armed with the *VR filtration*, $VR_1 \subseteq VR_2 \subseteq \ldots \subseteq VR_n$, we assess changes in topological summaries of the grid network. The idea is to detect long-lived (or persistent) features over wide range of thresholds $\nu_j$, and which might have impact on power grid network functionality. The commonly used topological summaries of persistent features are, for example, the *Betti numbers* and *persistent diagrams* [16, 19, 20]. For a given simplicial complex, Betti-0 ($\beta_0$) gives the number of its connected components; Betti-1 ($\beta_1$) is the number of one-dimensional holes, Betti-2 ($\beta_2$) is the number of two-dimensional holes etc. (see [21] on the detailed discussion on how to calculate Betti numbers). Finally, the persistent diagram gives the birth and death times of topological features in the $(x, y)$-coordinate system, where $x$-coordinate represents the birth time and $y$-coordinate represents the death time in filtration [22].

**Topological features under attacks** Typically vulnerability of a network under attacks is measured on the basis of the remaining global topological feature, such as, e.g, connectivity, largest subgraph size, and average path length, after each node removed according to a selected attack strategy. Recently, [10] considered changes in local geometry of power grids, characterized by motifs, under different node attacks. In turn, [9] introduced analysis of Betti numbers under node-based attacks and electrical conductance as edge weight. In this study our goal is to analyze utility of persistent topological features, e.g., Betti numbers $\beta_p$, $p \in Z^+$, under both node- and edge-based attacks and various transmission line parameters as edge-weights.

Furthermore, we evaluate sensitivity of these topological summaries to the attack. Note that many of the introduced TDA concepts are abstract and may not have straightforward explanation on the relations between, e.g., the number of $p$-dimensional holes and the power system robustness. Our main postulate here is that a system can be called more resilient if it tends to preserve longer its original properties under the attack, and our primary focus is on analysis of geometric properties of the system. In turn, the Betti numbers $\beta_p$, $p \in Z^+$, and persistent diagrams provide systematic characterization of the system geometry. As a result, power grid networks can be classified in terms of their resilience to the attack at a multi-scale level.

Algorithm 1 gives a brief an description of how distribution of Betti numbers ($\beta_k$, $k = 0, 1$) are calculated under edges based attacks. The method can be easily modified for different node based attacks described in [10, 23].

**Algorithm 1:** Edge based attack tolerance of networks based on topological features.

**Input** : Power grid G=(V, E, $\omega$).
**Output** : Changes in the Betti numbers under attacks relative to the initial Betti numbers prior to the attack.

$\vec{\beta}_k^{or} = \{\beta_{k1}^{or}, ..., \beta_{kn}^{or}\}$, - the sequences of Betti numbers from VR complex filtration before the attack for $k = 0, 1$ ;
$||.||_2$- Euclidean distance ;
$\omega' \leftarrow$ sorted $\omega$ (descending) ;
**for** $i = 1$ **to** $|E'|$ **do**
  $E_{remove} = \{(u, v) \in E : \omega(u, v) = \omega'(i)\}$ ;
  $E = E - E_{remove}$ ;
  Conduct VR complex filtration and calculate $\vec{\beta}_k = \{\beta_{k1}, ..., \beta_{kn}\}$ ;
  calculate normalized changes in the Betti numbers: $\Delta_k[i] = ||\vec{\beta}_k^{or} - \vec{\beta}_k||_2 / ||\vec{\beta}_k^{or}||_2, \quad k \in \{0, 1\}$ ;
**end**;

**TDA based network similarity measures** We can compare two power grid networks $G_1$ and $G_2$ by assessing their persistence diagrams $D_1$ and $D_2$, respectively [13, 20]. A commonly used distance measure is Wasserstein distance [24] which can be defined as

$$W_r(D_1, D_2) = \inf_{\gamma} \Big( \sum_{x \in D_1} ||x - \gamma(x)||_\infty^r \Big)^{1/r},$$

where $\gamma$ ranges over all bijections between $D_1$ and $D_2$, and $||z||_\infty = \max_i |z_i|$.

## 3. DATA

A critical motivation of this study is to apply the proposed method to real-world power systems with high renewable penetration. However, due to confidentiality requirements on critical infrastructure information, data availability of real-world power systems on actual scale is usually limited. Therefore, to simulate real-world power system operation, we use a well developed, publicly available synthetic network developed by Birchfield et al. in [25].

This study focuses only on the transmission network, which bridges the generators and distribution systems via substations. The transmission networks are often operated at tens or even hundreds of kilovolts to reduce transmission loss over long distances. The Texas system used in this study is built on the footprint of the Electric Reliability Council of Texas (ERCOT), which is the independent system operator that serves most of the Texas territory. As suggested in the previous section, each edge in the graph is associated with a weight ($\omega$)

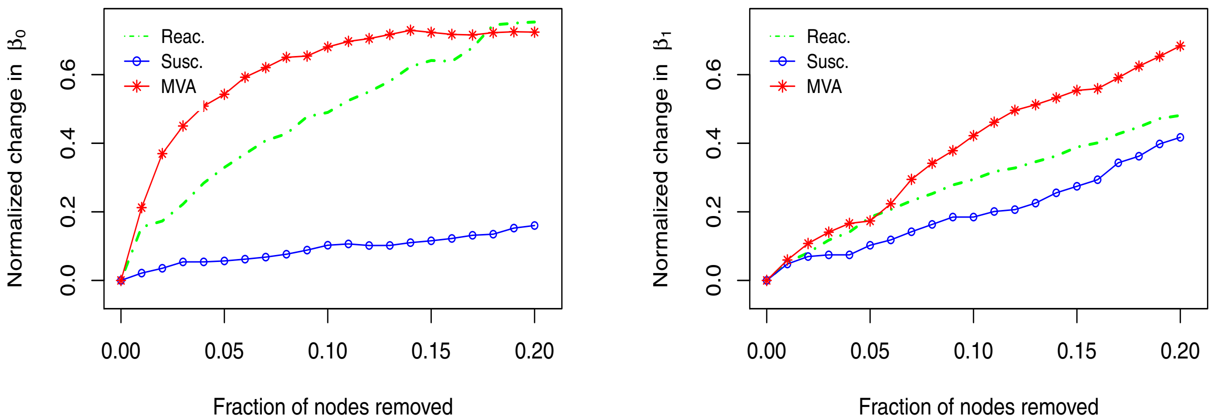


**Fig. 1***: Dynamics of the Betti numbers under node based attacks.*

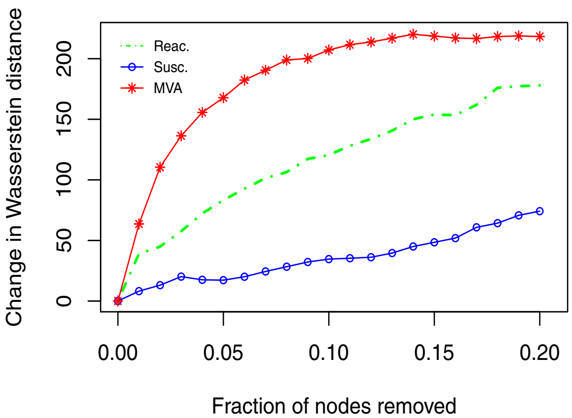


**Fig. 2***: Dynamics of the Wasserstein distances among the persistent diagrams, based on one-dimensional holes, under node based attacks.*

that measures the impact on grid robustness due to removal on the edge.

The study starts by investigating transmission line electrical parameters. Transmission line electrical parameters required for AC power flow analysis commonly include series impedance, shunt admittance, and apparent power (Mega Volt Amp, MVA) limits. The series impedance and shunt admittance are given by a standard $\pi$ model [26], while the maximum MVA limits are proportional to the product of current ratings and voltage ratings. For simplicity, we only consider the reactance term in the series impedance by following assumptions in the DC power flow model [27], while the shunt admittance typically consists of susceptance terms only. Therefore, we repeat our analysis by using the series reactances, shunt susceptances, and MVA ratings as the edge weights.

## 4. RESULTS & DISCUSSION

As previously mentioned, we consider three different weights for the edges of the transmission lines. For nodes with no direct connection (*i.e,* edges), we set the weight $\omega_{uv}$ to infinity if $e_{uv} \notin E$. Following the construction of the power grid networks, the VR complex filtration is run on each respective weighted network. Next, we calculate the threshold-wise Betti numbers $\beta_0$ and $\beta_1$. This process is repeated for each of the original networks and the subsequent network after each attack. The normalized Betti number change ($\Delta_k$) is computed using Alg. 1.

As shown in Fig. 1, both $\beta_0$ and $\beta_1$ deviations present an increasing trend as the fractions of nodes removed increase. In addition, Fig. 2 also displays similar trends across all three networks. However, we also observe significant differences in the trends across the three weighted networks. For example, the relative change of $\beta_0$ increases sharply from 0 to over 0.20 in the MVA-weighted network when only 0.01 of nodes are removed; while in the susceptance-weighted network, the relative change of $\beta_0$ does not exceed 0.20 even after 0.20 of nodes are removed. In addition, the increasing rates of the relative change of $\beta_0$ in the reactance-weighted network fall in between the other two networks. Likewise, the trends of the relative changes of $\beta_1$ also present similar trends: when 0.20 of nodes are removed, $\beta_1$ of the MVA-weighted network deviates by over 0.60, while the susceptance-weighted network shows less than 0.40 of relative change. Overall we can add that the topological structure with susceptance as (edge) weights prevails longer than the other two weight functions.

The fact that the relative changes of both $\beta_0$ and $\beta_1$ increase at a significantly greater rates in the MVA-weighted network than the other two networks implies that, removing buses connected with transmission lines with higher MVA ratings tends to cause larger deformation to the topological structure. Specifically, since $\beta_0$ reflects the number of connected components in a network, our results suggest that it is more likely to divide the power grid into multiple disconnected parts by removing the buses connected with higher MVA-rated transmission lines. Again, we postulate that the removed nodes correspond to substations that are connected with long-distance, high voltage transmission lines. Since high voltage transmission lines often play a major role in

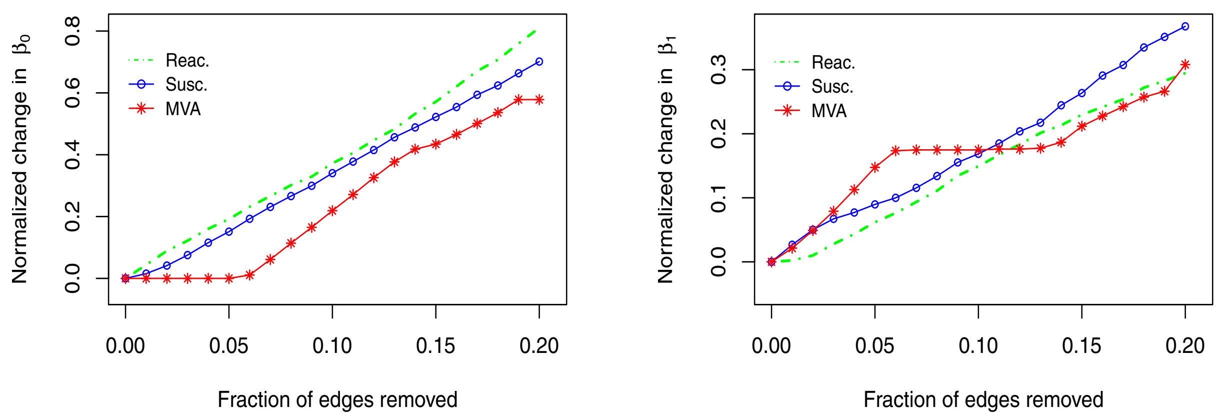


**Fig. 3***: Dynamics of the Betti numbers under edge based attacks.*

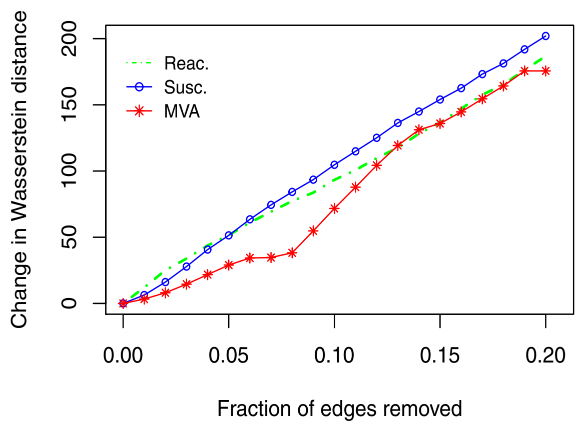


**Fig. 4***: Dynamics of the Wasserstein distances among the persistent diagrams, based on one-dimensional holes, under edge based attacks.*

transferring large amount of electric energy, it is critical to place emphasis on protecting these lines to meet power system resilience standard, and failures of these lines can lead to outage over large areas. Likewise, since branch reactances are determined by produce of branch lengths and unit reactance, a branch with high reactance can indicate either a long-distance transmission line, or a high unit reactance. In the former case, failure of the branch can also jeopardize the network security by splitting the network into isolated parts, which is implied by deformation of the topological structure, as indicated in Fig. 1. However, since the branch reactance is correlated with branch distance in a lesser extent than with the MVA ratings, the topological structure is less threatened by failure of nodes connected to lines with higher reactances than failure of notes connected to lines with higher MVA ratings. The targeting of nodes restructures the entire degree distribution (and thus the strength of nodes). In our style of edge attacks, we focus on removing fraction of edges depending on weight hierarchy (*i.e.,* from the highest down to the lowest). In Fig. 3, the normalized change in $\beta_0$ for the MVA-weighted network is unchanged until over 0.05 of the "weightiest" edges are removed. Comparatively, the reactance-weighted network exhibits steeper changes of $\beta_0$. For $\beta_1$, the MVA-weighted network record steeper $\beta_1$ changes as the fraction of removed edges rise up until 0.10, after which the susceptance-weighted network has the highest relative changes. For change in the wasserstein distance , as seen in Fig. 4, the resolve becomes more consistent with the happenings of the $\beta_0$ numbers. Furthermore, influence of the edge based attacks is found to substantially less affect the topology of the grid network, comparing to the node based attacks.

## 5. FUTURE WORK

In this study, we have assessed resilience properties of a simulated version of the Texas power grid network, from the perspective of topological data analysis and, particularly, persistent homology. In particular, we have evaluated dynamics and sensitivity of power grid topological summaries, namely, Betti numbers, under both node- and edge-based targeted attacks and the series reactances, shunt susceptances, and MVA ratings as the edge weights. The Texas power grid network appears to exhibit the highest fragility for the MVA weight function, and as expected the system topology remains more stable under edge-based attacks comparing to the node-based attacks. Our findings suggest that persistent homology of the power grid can be used as a system resilience indicator both at local and global levels.

In the future we plan to extend this analysis to power grid networks with multi-labeled nodes, i.e., when the attacker can focus on particular type of nodes such as power plants, as well as to assess vulnerability of the power system under integration of solar, wind, and distributed energy resources. Furthermore, we will investigate the relationship between global resilience measures based on functional grid properties, such as blackout size [7], and the newly introduced TDA resilience measures.

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